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By means of the equality (3), and analogues to (2) and (3)

$$\frac{CB'}{CF} = \frac{ca_2}{ca - c_2a_1}, \qquad \frac{A'B'}{CF} = \frac{c(a_1b_1c_1 - a_2b_2c_2)}{(ca - c_2a_1)(bc - b_2c_1)},$$

we deduce from (4)

$$\frac{\Delta''}{\Delta} = \frac{(a_1b_1c_1 - a_2b_2c_2)^2}{(bc - b_2c_1)(ca - c_2a_1)(ab - a_2b_1)}.$$

340 (Calculus) [1913, 196; 1919, 213]. Proposed by C. N. SCHMALL, New York, N. Y.

A pencil of parallel rays of light is incident upon a lens whose faces have the radii r_1, r_2, respectively. Show that the distance of the principal focus from the center of the first face of the lens will be a maximum or a minimum when

$$\frac{r_1}{r_2} = \frac{(\mu - 1)^{1/2}}{1 + (\mu - 1)^{1/2}},$$

where μ has its usual meaning.

SOLUTION BY H. S. UHLER, Yale University.

It is necessary to remark at the very beginning that the given result appears incorrect as it does not involve the thickness of the lens, and the statement of the problem is ambiguous since it does not specify whether the first or the second principal focus is meant.

By straightforward analysis, involving Snell's law, but too long to deserve publication in this place, I found

$$x = t + \frac{r_2}{\mu - 1} - \frac{\mu r_2^2}{(\mu - 1)[\mu(r_1 + r_2) - (\mu - 1)t]}, \tag{1}$$

where, to fix the ideas, a double convex lens was contemplated, $x \equiv$ distance from the center of the first (or incidence) face to the second principal focus (the focus beyond the emergence face), $t \equiv$ axial thickness of lens, $\mu \equiv$ index of refraction of the material (glass) of the lens with respect to that of the surrounding medium (air), and r_1 and r_2 denote the arithmetical values of the radii of curvature of the first and second faces of the lens, respectively.

Formula (1) may be checked by referring to James P. C. Southall's *The Principles and Methods of Geometrical Optics*, either edition, page 275, equation (170); and by changing his d, n, and r_2 to t, μ, and $-r_2$, respectively.

Now what quantity is to be the independent variable? Let it be assumed that t is to vary while μ, r_1, and r_2 remain constant. Then the necessary condition for a maximum or a minimum is

$$\frac{dx}{dt} = 1 - \frac{\mu r_2^2}{[\mu(r_1 + r_2) - (\mu - 1)t]^2} = 0,$$

which leads directly to

$$t = \frac{\sqrt{\mu}[r_1\sqrt{\mu} + r_2(\sqrt{\mu} \pm 1)]}{\mu - 1}. \tag{2}$$

To determine which sign has physical meaning we may proceed as follows: $x - t$ must not be negative, otherwise the principal focus will fall inside the material of the lens. Substituting in (1) the expression for $(\mu - 1)\,t$ given by (2) we find

$$x - t = \frac{r_2}{\sqrt{\mu} - 1} \quad \text{or} \quad -\frac{r_2}{\sqrt{\mu} + 1},$$

according as the upper or lower sign in (2) be taken. Since, under all ordinary conditions, $\mu > 1$ the upper sign alone is acceptable.

Continuing,

$$\frac{d^2x}{dt^2} = -\frac{2\mu(\mu - 1)r_2^2}{[\mu(r_1 + r_2) - (\mu - 1)t]^3}$$

which, for the critical value given by (2), reduces to

$$\left(\frac{d^2x}{dt^2}\right)_2 = +\frac{2(\mu - 1)}{r_2\sqrt{\mu}}$$

Since this result is positive the thickness given by (2) corresponds to a minimum focal length, as might be anticipated from purely physical considerations.

It is thus clear that a reasonable interpretation of the statement of the problem leads to an analytical condition which is quite different from the given answer. The thickness of the lens cannot be considered negligible (as is often done in very elementary discussions of problems in geometrical optics) because the numerator of (2) cannot vanish when the upper sign is taken. In other words, t cannot be avoided in the final condition.

In conclusion, attention may be called to the fact that each of $\partial x/\partial r_1 = 0$, $\partial x/\partial r_2 = 0$, $\partial x/\partial \mu = 0$ leads to a result which bears no relation to the given answer and hence t is the only sensible independent variable.

349 (Calculus) [1913, 312]. Proposed by C. N. SCHMALL, New York City.

If $y = a \cos(\log x) + b \sin(\log x)$, eliminate the constants a and b and obtain the equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0.$$

SOLUTION BY T. E. MERGENDAHL, Tufts College.

Taking the derivative of y, as given in the problem, and multiplying the result by x, we have

$$x\frac{dy}{dx} = -a \sin\log x + b\cos\log x.$$

Applying the same process to both sides of this equation, we find

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -(a\cos\log x + b \sin\log x) = -y,$$

or

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0.$$

Also solved by W. W. BEMAN, T. M. BLAKSLEE, P. J. DA CUNHA, H. T. DAVIS, H. H. DOWNING, G. H. GRAVES, P. HANSEN, A. M. HARDING, W. L. MISER, A. PELLETIER, ELIJAH SWIFT, and H. S. UHLER.

385 (Calculus) [1915, 161; 1919, 73]. Proposed by H. B. PHILLIPS, Massachusetts Institute of Technology.

If $f(x)$ is continuous between a and x, show that

$$\lim_{n \doteq \infty} \frac{\underline{|n}}{(x-a)^n}\int_a^x \cdots \int_a^x f(x)dx^n = f(a).$$

T. H. GRONWALL, New York City, offers the following criticism and completion of the solution already published.

First, the passage to the limit for $n \doteq \infty$, which forms the last step (p. 74), requires justification (this is however easily done by using Taylor's theorem with remainder term). Second, the problem as proposed assumes only that $f(x)$ is continuous, so that the use of the *derivatives* of $f(x)$ is not permissible, since they may not exist. The following proof requires $f(x)$ to be bounded in the interval from a to x and to be continuous only at the point a, but not in the whole interval. It is readily shown (integration by parts and complete induction) that

$$\int_a^x \cdots \int_a^x f(x)dx^n = \frac{1}{(n-1)!}\int_a^x (x-\xi)^{n-1}f(\xi)d\xi;$$[1]

writing $\xi = a + t(x-a)$ and $f(\xi) = \varphi(t)$, the formula to be proved becomes

$$\lim_{n\doteq\infty} n\int_0^1 (1-t)^{n-1}\varphi(t)dt = \varphi(0)$$

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[1] A proof of this relation may be found in Goursat-Hedrick's *Mathematical Analysis*, Vol. 2, pt. II, p. 36.—EDITORS.